

## ON A QUESTION OF ZADROZNY

MOHAMMAD GOLSHANI

In this short note, I will discuss the following question of Zadrozny [1].

**Question 0.1.** *Assume  $0^\sharp$  does not exist. Let  $M$  be a model of ZFC. Is there a model  $N$  of ZFC extending  $M$  with  $ON^N = ON^M$  such that  $HOD^N = L$ ?*

Our next result gives a negative answer to this question in a strong way.

**Theorem 0.2.** *Any model  $V$  of ZFC has a class generic extension  $V' \models \text{“ZFC”}$  such that if  $W \supseteq V'$  is a model of ZFC with  $ON^W = ON^{V'}$ , then  $HOD^W \neq V$ .*

*Proof.* Force over  $V$  by the reverse Easton iteration to add a new Cohen subset to each successor cardinal. Call the resulting extension  $V_1$ . By Jensen’s coding theorem,  $V_1$  has a class generic extension  $V_2$  such that for some real  $R \in V_2$ , we have  $V_2 \models \text{“}V = L[R]\text{”}$ . We show that  $V' = V_2$  is as required. Thus suppose  $W \supseteq V'$  is a model of ZFC, and suppose on the contrary that  $HOD^W = V$ . Then as  $R \in W$ , by a result of Vopenka,  $R$  is set generic over  $HOD^W = V$ , and this is a contradiction.  $\square$

## REFERENCES

- [1] Zadrozny, Włodzimierz Iterating ordinal definability. *Ann. Pure Appl. Logic* 24 (1983), no. 3, 263-310.

School of Mathematics, Institute for Research in Fundamental Sciences (IPM), P.O. Box: 19395-5746, Tehran-Iran.

E-mail address: golshani.m@gmail.com